EXERCISE – IV ADVANCED SUBJECTIVE QUESTIONS

1. $\frac{dy}{dx} - y\,\ell n\,2 = 2^{\sin x}.(\cos x - 1)\ln 2$, y being bounded when $x \to +\infty$.

2. $\frac{dy}{dx} = y + \int_0^1 y\,dx$ given y = 1, where x = 0

3. Given two curves y = f(x) passing through the points (0, 1) & $y = \int_{-\infty}^{x} f(t)\,dt$ passing through the points (0, 1/2). The tangents drawn to both curves at the points with equal abscissas intersect on the x-axis. Find the curve f(x).

4. Consider the differential equation, $\frac{dy}{dx} + P(x)y = Q(x)$

(i) If two particular solutions of given equation u(x) and v(x) are known, find the general solution of the same equation in terms of u(x) and v(x).

(ii) If α and β are constants such that the linear combinations $\alpha\cdot u(x) + \beta\cdot v(x)$ is a solution of the given equation, find the relation between α and β.

(iii) If w(x) is the third particular solution different from u(x) and v(x) then find the ratio $\frac{v(x)-u(x)}{w(x)-u(x)}$.

5. Find the curve which passes through the point (2, 0) such that the segment of the tangent between the point of tangency & the y–axis has a constant length equal to 2.

6. $x\,dy + y\,dx + \frac{x\,dy - y\,dx}{x^2+y^2} = 0$

7. $\frac{y\,dx - x\,dy}{(x-y)^2} = \frac{dx}{2\sqrt{1-x^2}}$, given that y = 2 when x = 1

8. Find the equation of the curve passing through the origin if the middle point of the segment of its normal from any point of the curve to the x-axis lies on the parabola $2y^2 = x$.

9. Find the continuous function which satisfies the relation, $\int_0^x t\,f(x-t)\,dt = \int_0^x f(t)\,dt + \sin x + \cos x - x - 1$, for all real number x.

10. $(1 - x^2)^2 dy + \left(y\sqrt{1-x^2} - x - \sqrt{1-x^2}\right)dx = 0$.

11. $3x^2y^2 + \cos(xy) - xy\sin(xy) + \frac{dy}{dx}\{2x^3y - x^2\sin(xy)\} = 0$.

12. Find the integral curve of the differential equation, $x(1 - x\,\ell ny).\frac{dy}{dx} + y = 0$ which passes through $\left(1, \frac{1}{e}\right)$.

13. Find all the curves possessing the following property; the segment of the tangent between the point of tangency & the x-axis is bisected at the point of intersection with the y-axis.

14. A perpendicular drawn from any point P of the curve on the x-axis meets the x-axis at A. Length of the perpendicular from A on the tangent line at P is equal to 'a'. If this curve cuts the y-axis orthogonally, find the equation to all possible curves, expressing the answer explicitly.

15. A curve passing through (1, 0) such that the ratio of the square of the intercept cut by any tangent off the y-axis to the subnormal is equal to the ratio of the product of the co-ordinates of the point of tangency to the product of square of the slope of the tangent and the subtangent at the same point. Determine all such possible curves.

16. A & B are two separate reservoirs of water. Capacity of reservoir A is double the capacity of reservoir B. Both the reservoirs are filled completely with water, their inlets are closed and then the water is released simultaneously from both the reservoirs. The rate of flow of water out of each reservoir at any instant of time is proportional to the quantity of water in the reservoir at that time. One hour after the water is released, the quantity of water in reservoir A is 1.5 times the quantity of water in reservoir B. After how many hours do both the reservoirs have the same quantity of water ?

17. A tank consists of 50 litres of fresh water. Two litres of brine each litre containing 5 gms of dissolved salt are run into tank per minute; the mixture is kept uniform by stirring, and runs out at the rate of one litre per minute. If 'm' grams of salt are present in the tank after t minute, express 'm' in terms of t and find the amount of salt present after 10 minutes.

18. Let $f(x, y, c_1) = 0$ and $f(x, y, c_2) = 0$ define two integral curves of homogeneous first order differential equation. If P_1 and P_2 are respectively the points of intersection of these curves with an arbitrary line, $y = mx$ then prove that the slopes of these two curves at P_1 and P_2 are equal.

19. Find the curve for which the portion of y-axis cut-off between the origin and the tangent varies as cube of the abscissa of the point of contact.

20. Find the orthogonal trajectories for the given family of curves when 'a' is the parameter.
(i) $y = ax^2$ (ii) $\cos y = a\,e^{-x}$ (iii) $x^k + y^k = a^k$
(iv) Find the isogonal trajectories for the family of rectangular hyperbolas $x^2 - y^2 = a^2$ which makes with it an angle of 45º.